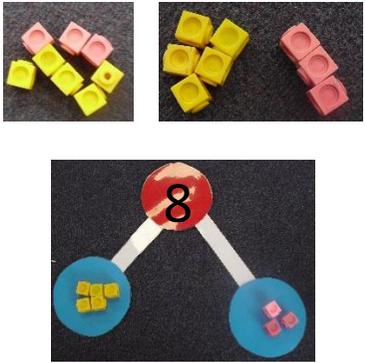
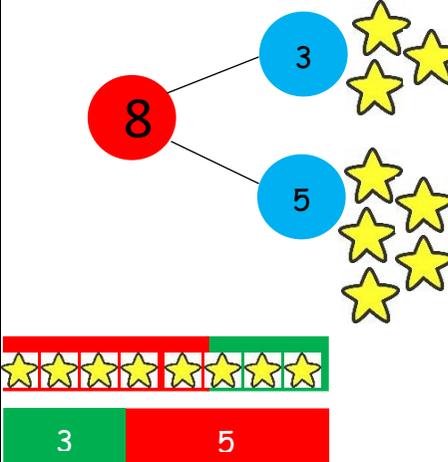
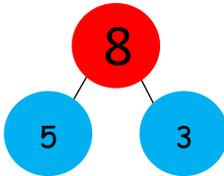
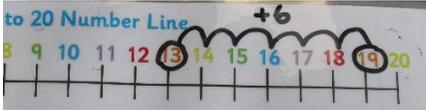
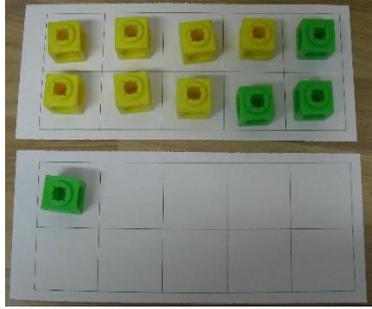


Progression of Skills

Addition

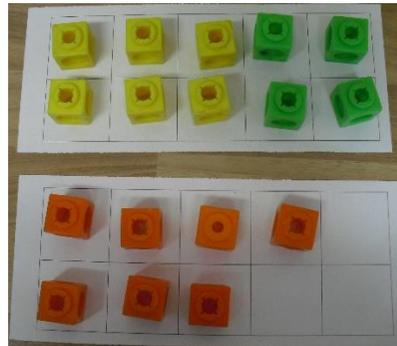
Key Concept	Concrete	Pictorial	Abstract
<p>Combining 2 parts together to make a whole (part, part, whole model).</p>	 <p>Using cubes to represent two groups and adding them together.</p>	 <p>$5 + 3 = 8$</p> <p>Using pictures to add two numbers together. Also supported through a bar model representation.</p>	<p>$5 + 3 = 8$ $8 = 3 + 5$</p>  <p>Using the part-part whole diagram to move into a more abstract approach.</p>

Progression of Skills

<p>Starting at the bigger number and counting on.</p>	 <p>Using cubes, start with the larger number and then count on the smaller number in ones to find the answer.</p>	<p>$13 + 6 = 19$</p>  <p>Circle the largest number and count on the smaller number in ones.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find the answer.</p>
<p>Regrouping to make 10.</p>	<p>$7 + 4 = 11$</p>  <p>Start with the bigger number and use the smaller number to make 10.</p>	<p>$7 + 4 = 11$</p>  <p>Using pictures, regroup or partition the smaller number to make 10.</p>	<p>$7 + 4 = 11$</p> <p>Children to think if they are at 7, how many more do they need to add to 10. Then how many do I add on?</p>

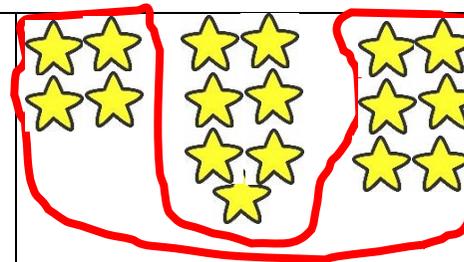
Progression of Skills

Adding 3 single digits.



$$4 + 7 + 6 = 17$$

Put the 4 and 6 together to make 10. Then add on the 7.



$$4 + 7 + 6 = 17$$

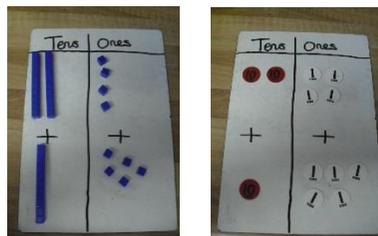
Draw a picture representation of the question. Combine the 4 and 6 together to make 10. Then add on the 7.

$$\begin{aligned} 4 + 7 + 6 &= 10 + 7 \\ &= 17 \\ &\quad 10 \end{aligned}$$

Combine the two numbers that make 10 and then add on the remainder.

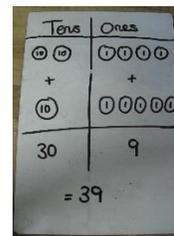
Column method without regrouping.

$$24 + 15 = 39$$



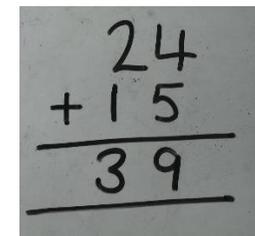
Using Base 10 or place value counters (dienes), add together the ones first before adding the tens.

$$24 + 15 = 39$$



When finished using resources, children should draw the place value counters to help them solve addition equations.

$$24 + 15 = 39$$

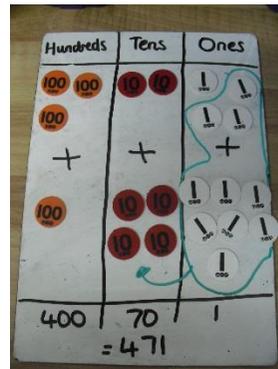


Children should use the column method to help solve their addition equations.

Progression of Skills

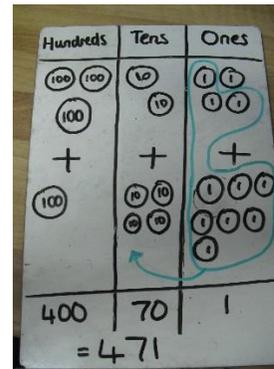
Column method with regrouping.

$$324 + 147 = 471$$



Using place value counters, children to make the two numbers and place them on a place value grid.

$$324 + 147 = 471$$



Children can draw a pictorial representation of the columns and place value counters to help them solve the addition equations.

$$324 + 147 = 471$$

$$324 + 147 =$$

$$300 + 100 = 400$$

$$20 + 40 = 60$$

$$4 + 7 = 11$$

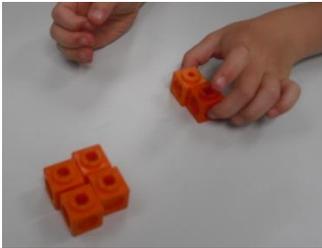
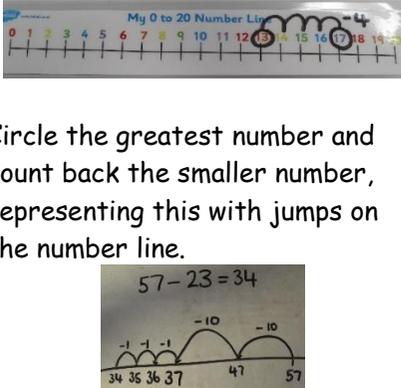
$$400 + 60 + 11 = 471$$

$$\begin{array}{r} 324 \\ + 147 \\ \hline 471 \\ \hline \end{array}$$

Children to partition the numbers before using the column method, to help them understand the exchange.

Progression of Skills

Subtraction

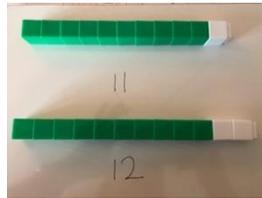
Key Concept	Concrete	Pictorial	Abstract
Taking away ones.	$6 - 2 = 4$  Use physical resources to show that objects can be taken away.	$6 - 2 = 4$  Cross out pictorial representations to show what has been taken away.	$6 - 4 = 2$ $18 - 3 = 15$ $18 = 20 - 2$
Counting back.	$17 - 4 = 13$  Make the larger number in your subtraction equation. Take them away slowly, counting backwards in ones as you go.	$17 - 4 = 13$  Circle the greatest number and count back the smaller number, representing this with jumps on the number line. Children can also begin to draw their own number line to solve the equation.	$17 - 4 = 13$ Put 17 in your head and count back 4. What number have you arrived at?

Progression of Skills

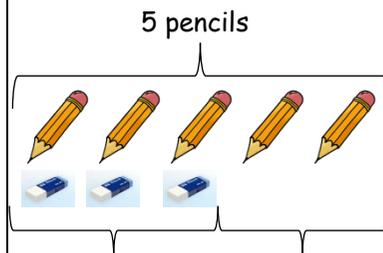
Finding the difference.

$$12 - 11 = ?$$

Compare amounts and objects to find the difference.



Use cubes to build towers or make bars to find the difference.

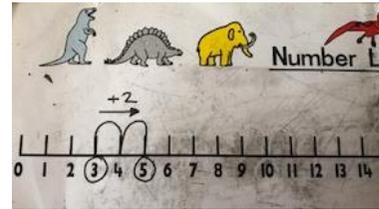


3 erasers

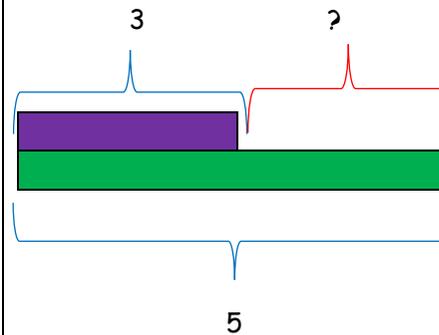
?

Use a basic bar model with objects to find the difference.

$$5 - ? = 3$$



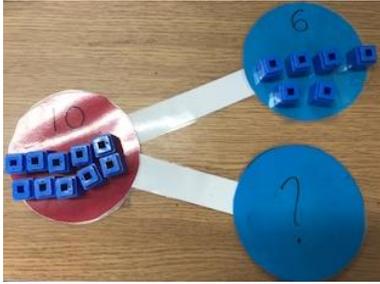
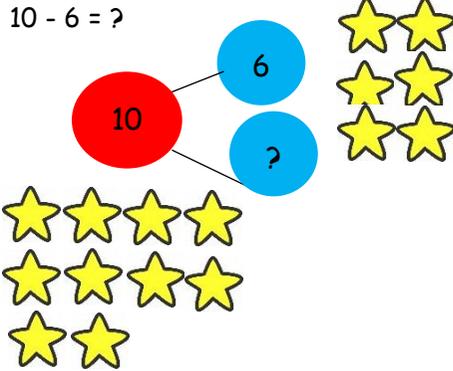
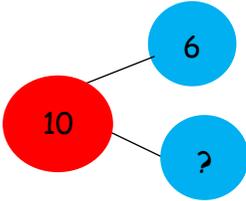
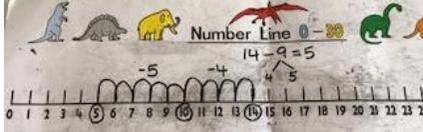
Count on to find the difference.



Draw a bar model to find the difference between two numbers.

Hannah has 23 sandwiches and Helen has 15 sandwiches. Find the difference between the number of sandwiches.

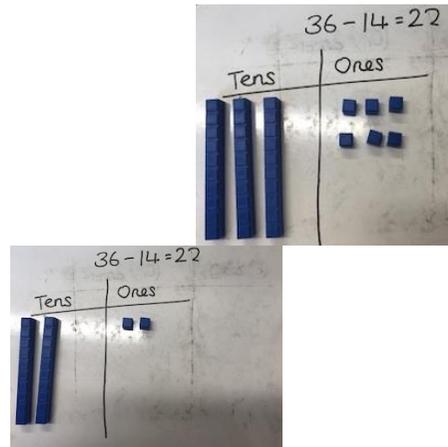
Progression of Skills

<p>Part-Part Whole Model.</p>	<p>$10 - 6 = ?$</p>  <p>Use a part-part whole diagram to help explain the inverse between addition and subtraction.</p> <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p>	<p>$10 - 6 = ?$</p>  <p>Use a pictorial representation of objects to show the part-part whole model.</p>	<p>$10 - 6 = ?$</p>  <p>Use numbers within your part-part whole model.</p>
<p>Making 10.</p>	<p>$14 - 5 = ?$</p>  <p>Make 14 on the tens frame. Take away the 4 first to make 10 and then take away one more so you have taken away 5. You are left with the answer 9.</p>	<p>$14 - 9 = ?$</p>  <p>Start at the 14. Take away 4 to reach 10. Then take away the remaining 5 so you have taken away 9 altogether. You have reached your answer.</p>	<p>$14 - 9 = ?$</p> <p>How many do we take off to reach the next 10? How many do we have left to take off?</p>

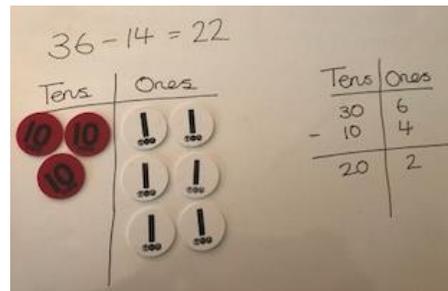
Progression of Skills

Column method without regrouping.

$$36 - 14 = ?$$

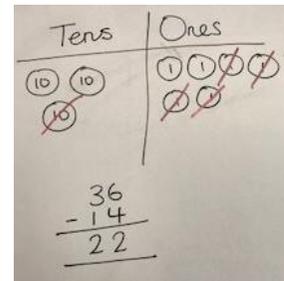
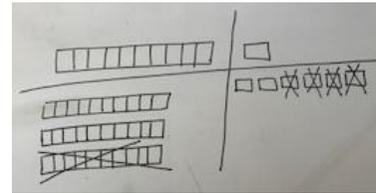


Use Base 10 (dienes) to make the bigger number and then take away the smaller number.



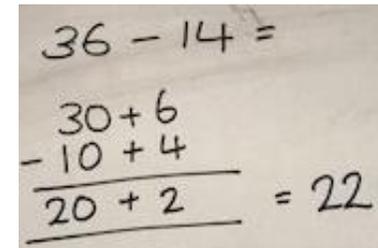
Show how you partition numbers to subtract. Make the larger number first.

$$36 - 14 = ?$$

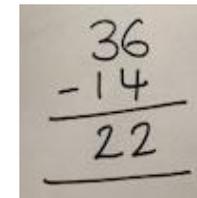


Draw the Base 10 (dienes) or use place value counters alongside the written calculation to help show working.

$$36 - 14 = ?$$



This will lead to a clear written column subtraction.



Progression of Skills

Column method with regrouping.

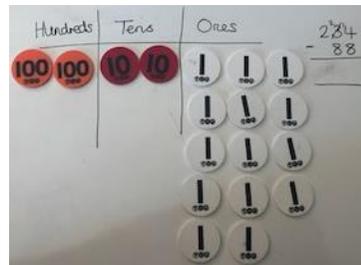
$$234 - 88 = ?$$

Use Base 10 (dienes) to start with before moving onto place value counters. Start with one exchange before moving onto subtraction with two exchanges.

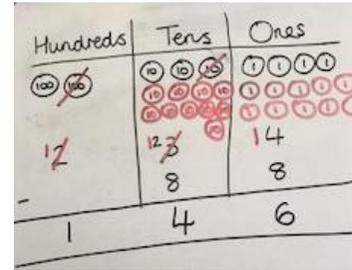
Make the larger number with the place value counters.



Start with the ones, can take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

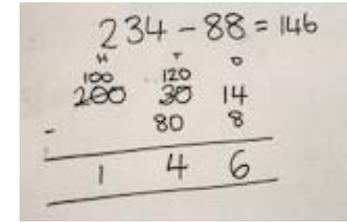


$$234 - 88 = ?$$

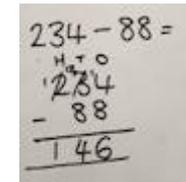


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

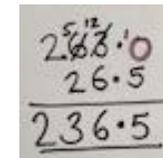
$$234 - 88 = ?$$



Children can start their formal written method by partitioning the number into clear place value columns.



Once children are confident with this they can use a more compact method.



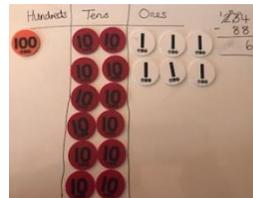
This will lead the children into an understanding of subtracting any number, including decimals.

Progression of Skills

Now I can subtract my ones.



Then look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



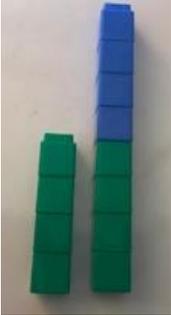
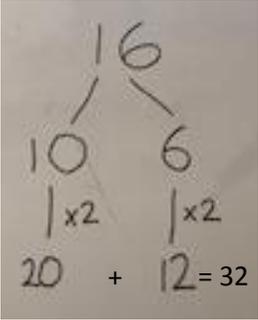
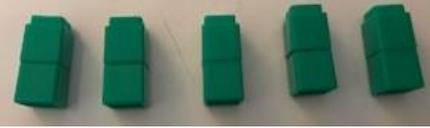
Now I can take away my eight tens and complete my subtraction.



Show the children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

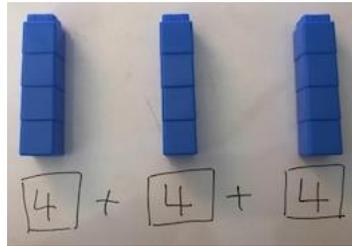
Progression of Skills

Multiplication

Key Concept	Concrete	Pictorial	Abstract
Doubling.	 <p>Using a range of practical resources to show how to double a number.</p>	<p>Double 4 is 8.</p>  <p>Draw pictures to represent how to double a number.</p>	 <p>Partition a number and then double each part before you recombine it back together.</p>
Counting in Multiples.	 <p>Count in multiples which are supported by concrete objects in equal groups.</p>	  <p>Use a number line or pictures to continue to support children in counting in multiples.</p>	<p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p> <p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p>

Progression of Skills

Repeated Addition.

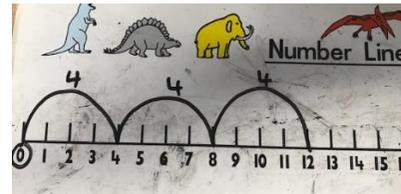


Use different objects to add equal groups.

There are 3 plates. Each plate has 2 star biscuits on it. How many biscuits are there?



2 add 2 add 2 equals 6.



$$4 + 4 + 4 = 12$$

Use a number line to support children's understanding.

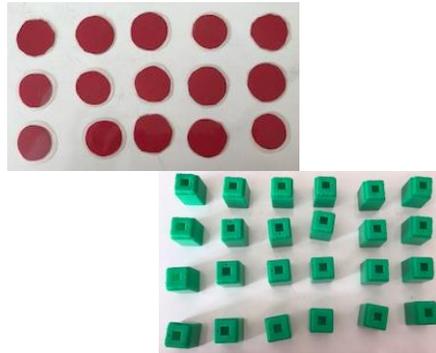


$$2 + 2 + 2 + 2 + 2 = 10$$

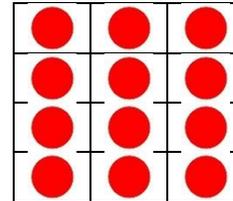
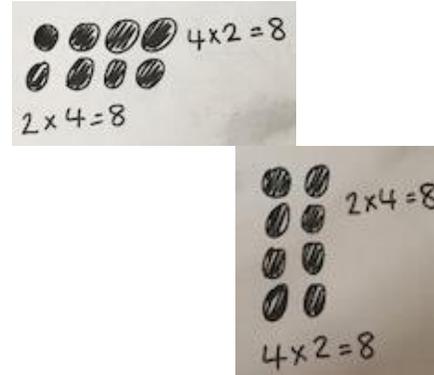
Write addition sentences in order to describe objects and pictures.

Progression of Skills

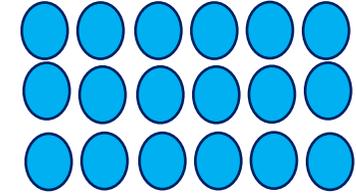
Arrays Showing Commutative Multiplication



Children to create a range of arrays using resources (counters/cubes) to represent multiplication sentences.



Get children to draw arrays in different rotations to find commutative multiplication sentences.



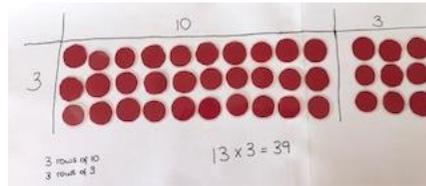
$$6 + 6 + 6 = 18$$
$$3 + 3 + 3 + 3 + 3 + 3 = 18$$
$$6 \times 3 = 18$$
$$3 \times 6 = 18$$

Use arrays to help children represent and write multiplication sentences and reinforce their understanding of repeated addition.

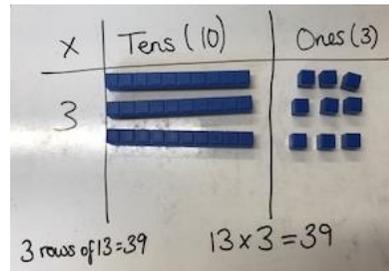
Progression of Skills

Grid Method.

Represent the link with arrays to first introduce the grid method.

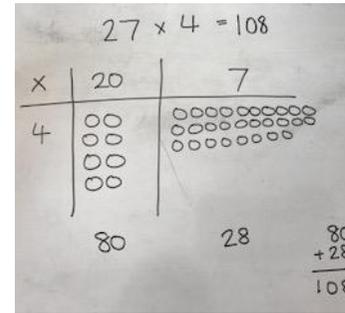


Move onto using Base 10 (dienes) resources to introduce a more compact method.



Then move onto using place value counters to show how we are finding groups of a number.

$$126 \times 4 =$$



Children can represent their understanding by using place value counters. They can draw the counters, use colours to represent the amounts or just draw circles in the different columns to represent their thinking.

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

X	40	5
4	160	20

$$160 + 20 = 180$$

Then, multiply by a 2 digit number showing the different rows within the grid method.

X	10	5
20	200	100
3	30	15

$$300 + 100 + 30 + 15 = 445$$

X	1,000	300	40	2
10	10,000	3,000	400	20
8	8,000	2,400	320	16

$$10,000 + 8,000 + 3,000 + 2,400 + 400 + 320 + 20 + 16 = 24,156$$

Progression of Skills

We are multiplying by 4 so we need 4 rows.

Hundreds	Tens	Ones	$126 \times 4 =$

Fill each row with 126.

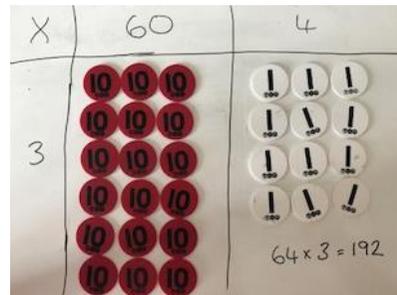
Hundreds	Tens	Ones	$126 \times 4 =$
100	10 10	1 1 1 1 1 1 1 1	
100	10 10	1 1 1 1 1 1 1 1	
100	10 10	1 1 1 1 1 1 1 1	
100	10 10	1 1 1 1 1 1 1 1	

$400 + 80 + 24 = 504$

Add up each column, starting with the ones in order to make any exchanges necessary.

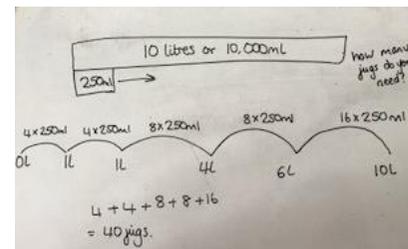
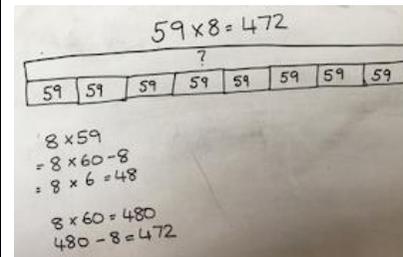
Progression of Skills

Column Multiplication.

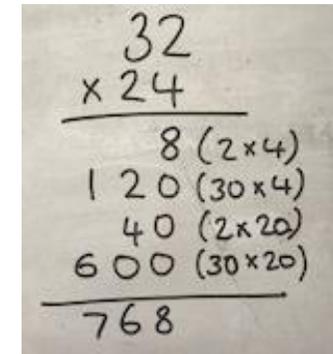


Children can continue to be supported by place value counters at this stage of multiplication.

It is important at this stage that they know to multiply the ones first and note down their answer following by the tens number which they also note down.



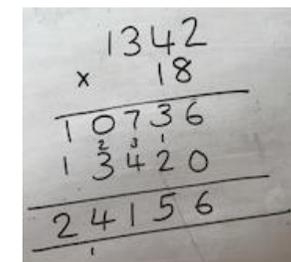
Introduce the bar model and number lines to support children when solving problems with multiplication alongside formal written methods.



Use long multiplication, making sure children understand that they need to clearly line up their numbers in columns.

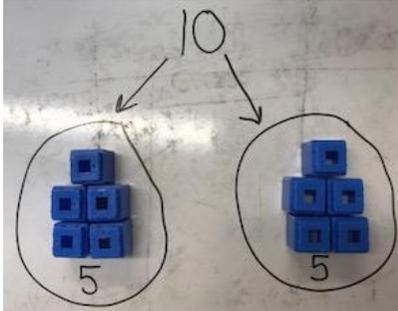
If it helps, children can write out the equation they are solving.

This then helps children understand the more compact method.



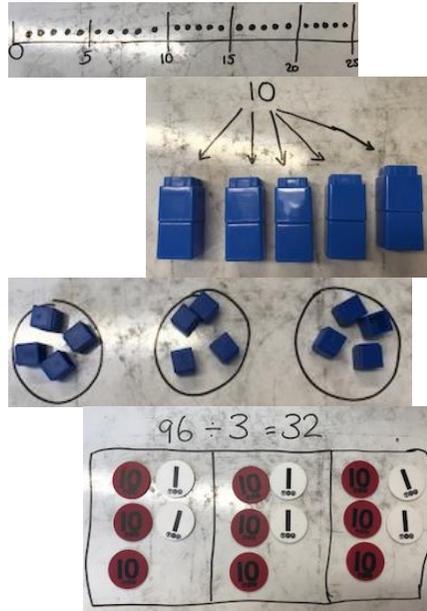
Progression of Skills

Division

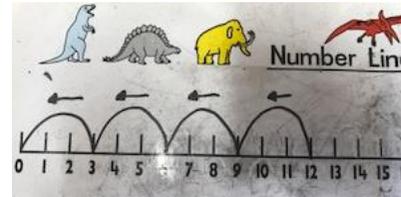
Key Concept	Concrete	Pictorial	Abstract
Sharing Objects Into Groups.	 <p>I have 12 cubes, can you share them equally into 2 groups?</p>	$8 \div 2 = 4$  <p>Children can use a range of pictures or shapes to share quantities.</p>	$12 \div 3 = 4$ Share 12 buns between three people.

Progression of Skills

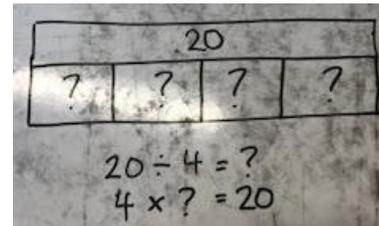
Division As Grouping.



Divide quantities into equal groups. Use a range of resources e.g. cubes, counters, objects or place value counters to support understanding.



Use a number line to represent the jumps in groups. The number of jumps equals the number of groups.



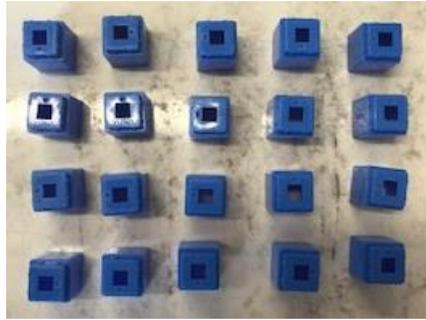
Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be in each group.

$$28 \div 7 = 4$$

Divide 28 into 7 groups. How many are in each group?

Progression of Skills

Division With Arrays.



Link division to children's knowledge of multiplication by creating an array and thinking about how number sentences can be created.

e.g. $20 \div 4 = 5$ $5 \times 4 = 20$
 $20 \div 5 = 4$ $4 \times 5 = 20$



Get children to draw an array and use lines to split this into groups to make the multiplication and division sentences.

$$6 \times 4 = 24$$

$$4 \times 6 = 24$$

$$24 \div 4 = 6$$

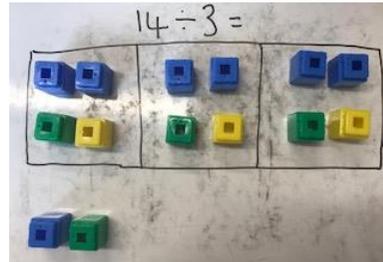
$$24 \div 6 = 4$$

Find the inverse of multiplication and division sentences by creating four linking number sentences.

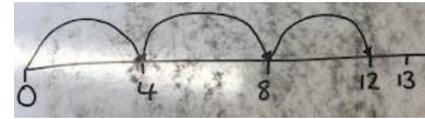
Progression of Skills

Division With A Remainder.

$$14 \div 3 = 4 \text{ remainder } 2$$

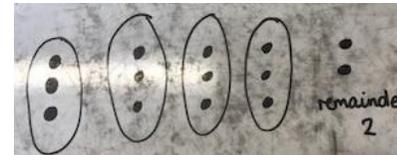


Divide concrete objects between groups and see how much is left over.

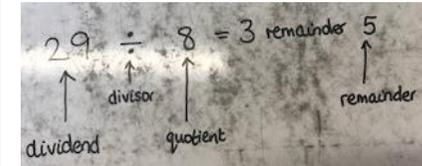


$$13 \div 4 = 3 \text{ r } 1$$

Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



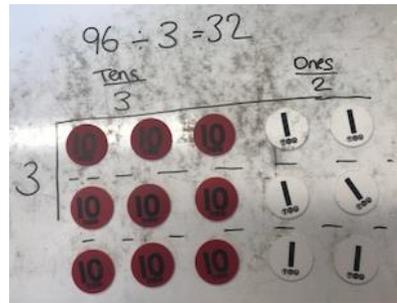
Draw dots and group them to divide an amount and clearly show a remainder.



Complete written division equations and show the remainder using the letter 'r'.

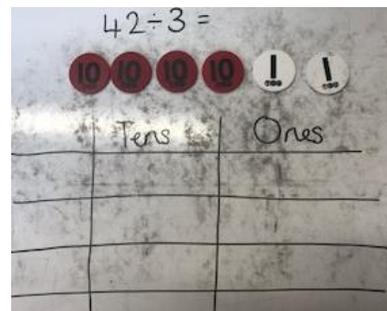
Progression of Skills

Short Division

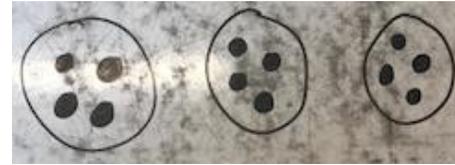


Use place value counters to divide using the bus stop method alongside.

$$42 \div 3 =$$

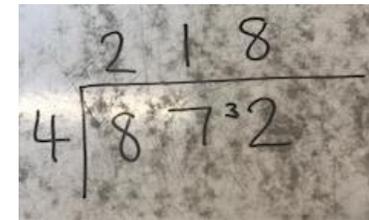


Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

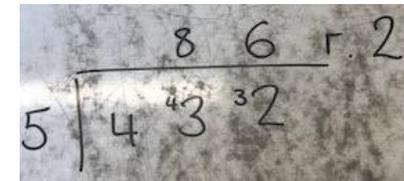


Get children to continue to use drawn diagrams with dots of circles to help them divide numbers into equal groups.

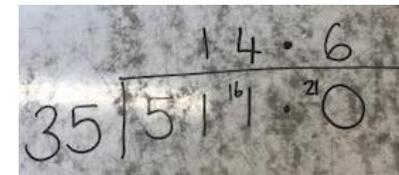
Encourage them to move towards counting in multiples to divide more efficiently.



Begin with divisions that divide equally with no remainder.



Move onto divisions with a remainder.

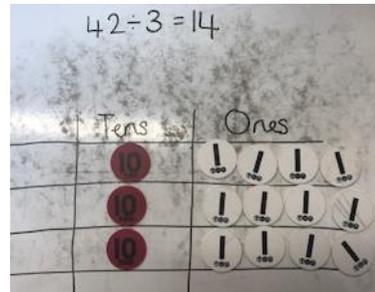


Finally move into decimal places to divide the total accurately.

Progression of Skills



We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much there are in 1 group so the answer is 14.